The Sales Counter at Arm-and-a-Leg Tickets: Does This Line Ever Move?

Mr. I. M. Boss is vice president in charge of operations for Arm-and-a-Leg Ticket Sales. He is concerned about complaints regarding long waits at the ticket windows on Friday afternoons at many of the malls. To reduce the number of complaints, Mr. Boss has hired Dr. Hye I. Cue, an expert on queueing theory.

Queueing theory deals with the mathematical study of waiting lines. The entities waiting in line can take on a variety of forms. They could be people waiting at a doctor's office or airplanes waiting on a runway. The waiting line is not even visible in every case. For example, telephone calls waiting for an operator are also "waiting in line. "

Before we begin to analyze Mr. Boss's problem, which is called a single-server model, we will make the following assumptions:

1. Individual customers arrive at random to purchase tickets.
2. The time to complete a purchase is also random. This might be due to the number of tickets the customer purchases or the customer asking for information about dates and seat location.

To use queueing theory, Dr. Cue needed to collect data about the customers. She spent several Friday afternoons observing the situation and collecting the data. She found that the average number of customers arriving per hour, a, is 18 and that the average number of customers the single ticket agent can help per hour, h, is 20.

1. What variables did Dr. Hye I. Cue observe during her Friday afternoon visits to the mall?

   ____

2. Dr. Cue observed that a = _________________________ and h = ________________________.

3. If 18 customers arrive per hour, on average, how much time occurs between successive arrivals?

   ____

4. If a customers arrive per hour, on average, how much time occurs between arrivals?

   ____

5. If 20 customers are helped per hour, on average, how long does it take to help one customer?

   ____

6. If h customers are helped per hour, on average, how long does it take to help one customer?

   ____

To develop a mathematical model of our queue, we must have an idea of the traffic intensity, x, which is the ratio of the average rate of customer arrivals, a, to the average rate of customers being helped, h. In order that this ratio make sense, the time units of a must be the same as those of h.

7. Write a ratio between the variables that represents the traffic intensity: x = __________. Using the values of a and h from question 2, in this case, x = ______________.

8. The average number of customers in the system, L, (including those in line and the one at the ticket window) can be represented by the function (x must lie between 0 and 1):

$$L = \frac{x}{1-x}$$

   Using the value of x from question 7, calculate L.______________________________

   What does this value of L tell you about this queueing system?_________________________

Customer satisfaction actually is more dependent upon the length of time it takes to get a ticket than the length of the line. Therefore, let:

W = the average time a customer waits in a system including the time in line and the time to be helped.

The function $L = aW$ expresses the relationship between L and W.

9. What are the units for each of the variables a, L, and W?

   ____

10. Given that $L = aW$, solve for W in terms of a and L.____________________________________

11. Using your values of a and L, calculate the value of W for the system at Arm-and-a-Leg Ticket Sales.

   W = ______________

   What do you know from this value of W?_________________________________

Suppose Dr. Cue learns that during a slow time of the day an average of only 16 customers per hour arrive to purchase tickets. However, the customer help (service) rate remains the same.

12. What is the traffic intensity, x, during this time of day? ______________________

13. Calculate the values of L and W for this time of day.

   L = __________________ W = ______________________

14. Enter the values of x, L, and W from questions 7, 8, 11, 12, and 13 in the appropriate places in the following table. Then complete the rest of the table for the given values of a and h.

a (customers/hr)	***h*** (customers/hr)	***x***	***L*** (customers)	***W*** (hours)
14	20			
16	20			
18	20			
18	22			
18	24			

15. Using the values you entered in the table above, what happened to the values of L and W when a increased while h remained constant?
____

16. What happened to the values of L and W when h increased while a remained constant? ____

17. If $a = 22$ and $h = 20$, what are the values of x, L, and W? x = ______ L = ______ W = ______

    Do all of these values make sense? _____ Explain. _______________________________________

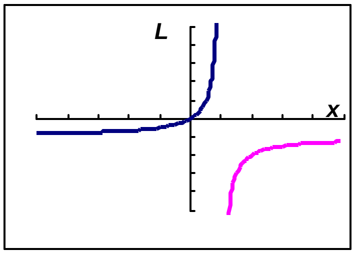
18. Why did this particular value of x occur? ____

19. Explain what would happen to the ticket line given the situation in number 17?
____
____

20. Compare the value of x in question 17 with all of the values of x in the table.

    What do you observe? __________________________________

The graph of $L = x/(1 - x)$ appears at the right.



21. Using the equation for L, what happens when $x = 1$?
____

22. Where is this represented on the graph?
____

23. For the function $L = x/(1 - x)$, what is the domain of x?
____

24. If $L = -1$, then $-1 = x/(1 - x)$. What happens when you solve this equation for x?
____

25. Where is this represented on the graph? ____

26. For the function $L = x/(1 - x)$, what is the range of L?
____

27. Referring to questions 17-20, what happens to the value of L when $x > 1$? ______________________

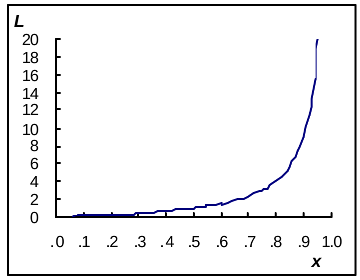
28. Discuss why such a value of L makes sense, or does not make sense, for the Arm-and-a-Leg problem.
____

29. Thinking about the definition of x, $x = a/h$, what would a value of $x = 0$ mean? ___________________

30. Trace the portion of the graph above that is appropriate for the Arm-and-a-Leg problem.

31. Write the domain for the portion of the graph you traced. ____________________________________

Calculate the values of *L* for each of the values of *x* in the table below. Enter the values of *L* into the table.



32.

x	***L***
0.8	
0.9	
0.95	
0.99	

33. What happens to *L* as the values of *x* approach (get closer to) $x = 1$? ____________________________

34. Where is this represented on the graph? ____

35. In all of the analyses above, changes were made in the values of *a* and *h*. In reality, which, if any, of these variables would a manager like Mr. I. M. Boss be able to control? ______________________

36. What could Mr. Boss do to improve customer satisfaction at his sales outlets? ____________________

____

____

37. What strategies to improve customer satisfaction have you experienced while waiting in line?

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