Fuel Blending at Jurassic Oil: How to Minimize the Cost of a Quality Blend

Teacher Resources

Operations Research

The field of Operations Research (OR) has its roots in the years just prior to World War II as the British prepared for the anticipated air war. In 1937 field tests started on what was later to be called radar. In 1938 experiments began to explore how the information provided by radar should be used to direct deployment and use of fighter planes. Until this time, the word experiment conjured up the picture of a scientist carrying out a controlled experiment in a laboratory. In contrast, the multi-disciplinary team of scientists working on this radar-fighter plane project studied the actual operating conditions of these new devices and designed experiments in the field of operations and the new term operations research was born. The team's goal was to derive an understanding of the operations of the complete system of equipment, people, and environmental conditions (e.g. weather, nighttime) and then improve upon it. Their work was an important factor in winning the Battle of Britain and operational research eventually spread to all of the military services. Several of the leaders of this effort were Nobel laureates in their original fields of study.

Their approach was later paralleled in the US, with the first team working on anti-submarine tactics. The US group developed a series of mathematical models entitled search theory that was used to develop optimal patterns of air search. Like their British counterparts, they got close to the action by riding in airplanes on patrol just as the modern operations researcher might ride in a police car or spend time in an automotive assembly plant. Currently, every branch of the military has its own operations research group that includes both military and civilian personnel. They play a key role in both long-term strategy and weapons development, as well as directing the logistics of actions such as Operation Desert Storm. In addition, the National Security Agency has its own Center for Operations Research.

OR moved into the industrial domain in the early fifties and paralleled the growth of computers as a business planning and management tool. As the field evolved, the core moved away from interdisciplinary teams to a focus on the development of mathematical models that can be used to model, improve, and even optimize real-world systems. These mathematical models include both deterministic models such as mathematical programming, routing or network flows and probabilistic models such as queuing, simulation and decision trees. These mathematical modeling techniques form the core curriculum in masters degree and doctoral programs in operations research which can be found in either engineering or business schools. Most mathematics departments also offer one or more introductory operations research courses at the junior or senior undergraduate level.

Mathematical Programming in the Petroleum Industry

The history of mathematical programming is closely intertwined with its application in the petroleum industry and the growth of computers. In the early 1950s papers were already being publishing on the potential of linear programming applications to fuel blending, as well as the scheduling of refineries. The simplest blending problem involved just one gasoline grade to be blended from three to ten individual components. An entire refinery might produce only four blends of gasoline out of 20 different components. Individual oil companies developed their own stored computer programs that were used on the early computers of the 1950s. A major problem was the limited storage of computers that restricted the size of the matrix that lies at the heart of the algebraic solution of linear programs.

The increase in problem size and the non-linearity of certain functions (e.g., the effect of lead on octane) motivated research into algorithms for solving increasingly complex problems. As the problems grew larger, they exceeded the capacities of computers. This led to development and implementation of algorithms that would split the problem into pieces and solve a series of linear programs. Analogous procedures were developed to approximate non-linearity. These new algorithms fueled the increase in usage of linear programming (LP) in the 1960s. In general, commercial LP packages did not exist at that time, and the user was still programming his own algorithms. During this time period, almost all medium and large petroleum companies developed refinery planning models using LP. By the end of the 1960s most companies began to realize that their newest challenge lay not with finding new and more efficient algorithms. The problem was how to manage the model with its large input data set and output that could run over 100 pages.

In the 1970s, companies such as Amoco, Shell and Exxon developed their own sophisticated in-house systems to manage the models. These systems included the concepts of data base management, matrix generation, and solution reporting in a compact form. During this time period with faster computers and the ability to model non-linear functions, the range of applications expanded. New models were developed that modeled the step-by-step process of converting crude to final products. These models supplemented the aggregate analysis used to determine which components to use in which blend.

The 1980s and 1990s have seen dramatic changes in computing power and standardization of software. The desktop user of computers can select from a large array of commercially available mathematical programming packages that can be linked to database managers or Excel spreadsheets. As a result, applications of mathematical programming have been able to move to lower and lower levels in the organizational hierarchy. In addition, these operational and planning models are becoming more and more linked to total enterprise models that are growing in their role of helping companies manage their entire supply chain. For example, Citgo developed an integrated products planning system. The system "helps top management with their weekly decisions concerning refinery run levels, where to buy and sell products and what prices to pay or charge, how much product to hold in inventory at each location and how much product to ship by each mode of transportation." The system is a powerful tool used by product managers, pricing managers, product traders and the budget manager in their financial, logistical and marketing planning. Lastly, the trend towards globalization is forcing companies to model not just a single plant or set of plants in one country but rather to begin integrating all of their plants around the world into one model. Du Pont is reported to have models that span its worldwide manufacturing capacity.

Abstracted primarily from "A History of Mathematical Programming in the Petroleum Industry", C. E. Bodington and T. E. Baker, INTERFACES 20:4 1990, pp. 117-127.

Objectives of the Module

The objective of this module is to motivate students to learn mathematics by demonstrating ways in which the mathematics they are learning is actually used in industry and government. Techniques of operations research such as linear programming are used to solve diverse problems in a wide variety of business and governmental settings.

The Jurassic Oil gasoline blending problem asks students to explore minimizing cost while still maintaining certain criteria for the blending of two distinct types of gasoline. It is an example of what are called "blending" problems, and it occurs whenever a company wants to manufacture a product with specific, unique characteristics, unlike any that is available, but can produce such a product by combining two or more products which are available. Students will learn how to use the graph of a system of linear inequalities to solve geometrically a basic problem involving only two decision variables. One way to extend the basic problem is to consider problems involving more than two decision variables. Although the geometric technique is no longer appropriate, the problem is still solvable. If the problem can be mathematically formulated, software packages can then be used to determine the optimal solution. Thus, the "coin of the realm" today and in the future in mathematics is problem formulation.

The specific objectives of the Jurassic Oil Blending module include:

1. Demonstrating the interdisciplinary nature of problem solving in industry; i.e., an understanding of chemistry, materials science, environmental science, and nutrition may be needed in setting up an appropriate system of constraints.
2. Introducing constraints that are determined using weighted averages.
3. Rewriting constraints with numeric right-hand sides.
4. Scaling the data to thousands of barrels (bbls).
5. Considering a feasible "region" which lies wholly on one straight line (the demand equality).

For Algebra 2 or higher level students, the objectives of the module also include: Providing a glimpse of **non-linear** programming. (For example, octane and gasoline volatility are non-linear functions which are *sometimes* approximated using linear functions.)

This module is intended to enhance student skill development by providing real-world examples to motivate student learning. The particular skills that are targeted in this module are:

1. Converting word problems into mathematical expressions
2. Finding weighted averages
3. Graphing linear equations
4. Graphing linear inequalities
5. Graphing systems of inequalities
6. Solving systems of linear equations and inequalities
7. Interpreting problem solutions

Initiating an Opening Dialogue

In order to motivate students for this lesson we have provided some discussion questions. Please take time to allow students to discuss **some** of these questions either in groups or as a class:

- How are different blends of gasoline created?
- How do you mix the various fillings (chicken, beef, pork, water) to produce a hot dog that is low in cost yet as high as possible in nutrition?
- How is gold mixed with various other metals to produce the different karat weights needed in designing and producing jewelry?
- How do you decide how much of each type of nut to mix in order to produce mixed nuts that are low in cost yet meet the public requests so that you can have maximum sales?

Using the Module

The base problem provided in this module is designed for students in an Algebra 2 course. In addition, an alternative scenario having fewer constraints and a two-dimensional feasible region is provided for students in an Algebra 1 course. The steps in formulating and solving a linear programming problem involve identifying the decision variables and an objective function, identifying any constraints on the decision variables, determining the feasible region, and locating the optimal solution in the feasible region. As students explore these steps in solving the problem, they should carefully consider the following definitions of the key concepts.

Decision variables are the variable quantities completely describing the decision to be made.

The two sample problems contain only two decision variables. This allows inequalities to be graphed in a coordinate plane, using graph paper or a calculator. In more complicated problems, where there are more than two decision variables, computer programs are needed to find the solutions.

Objective function is the quantity to be maximized or minimized which is defined in terms of the decision variables.

The objective function is the vehicle by which the decision maker maximizes or minimizes an amount. In the two sample problems, the goal is to minimize the total cost that Pete Troleum must pay the two companies to produce the grade of gasoline that he needs. The objective function is used to find this minimal cost.

Other optimization problems look to maximize an amount, looking for the largest value to the objective function. Students should be able to verbalize why maximization is not a goal of this problem. They should also be able to come up with examples when a person would want to maximize an objective function.

Constraints are restrictions on the values of one or more of the decision variables.

The constraints are linear inequalities and equations in the two sample problems. All the constraints include the line itself in its solution, so they will be solid lines when graphed.

Feasible region is the set of all points which satisfy all of the constraints.

It is the feasible region which will tell you all possible points to use in the objective function to find the minimal amount of total weekly gasoline purchase cost, in this module. The feasible region may be in the shape of a polygon (as in the Algebra 1 example). If equations are used along with inequalities, the feasible region could be a line segment(as in the Algebra 2 example) or even a point. All the points in the feasible region, including those on the boundary, satisfy all the constraints.

Optimal solution is the set of pairs of values of the decision variables which satisfy all of the constraints *and* achieve the goal of minimizing (or maximizing) the objective function.

Finding the optimal solution is usually done in two parts.

- locating a feasible region containing all of the points which satisfy all of the constraints;
- using the objective function to find the point in the feasible region which is optimal.

You may want to begin this process with students by posing questions such as:

"So how do you find the optimal solution for this problem? In other words, what is the minimal cost in order to buy the different grades of gasoline?"

Students need to understand that they have to find values of A and H that while minimizing the blending costs, nevertheless satisfy all of the constraints.

To ensure uniformity of solutions we have suggested that students graph A on the horizontal and H on the vertical axis. You may wish to discuss with the students the effect of graphing these variables the other way around or ask several students to do so.

In these problems it *appears* that A and H must be whole numbers, because parts of a barrel are apparently not sold. However, the "barrel" is a unit of measure. The amount of gasoline purchased may be delivered by a large tanker, or even via a pipeline, so that fractional barrels *could* be purchased. This may be an important question to discuss with students.

Once students have conceptualized the problem situation, the entire module could be conducted as a cooperative learning situation and/or a competition. Groups could be formed and named as competing oil companies. Each group could work at finding the smallest cost for the gasoline that meets all of the constraints.

Teaching Notes on Feasible Region (see **page 3** of student module)

1. Students are asked to graph the six constraints. You may want to discuss that these constraints are graphable because there are only two decision variables. In situations where there are more decision variables, it is necessary to use a computer to determine the solution.

2. When students are asked to graph the six constraints the issue of what scale to use should be discussed. You may wish to use the **coordinate** grid provided at the end of this Teacher Resource Guide.

3. After graphing the six constraints students should notice that the feasible region is a line segment. Most linear programming problems result in feasible regions that are polygons. Since one of the six constraints in this case is an equation, the region is reduced to a line segment.

4. Two of the six constaints, $A \leq 90{,}000$ and $H \leq 90{,}000$, are not boundaries of the feasible region. A discussion topic could be: For what values of N would $A \leq N$ and $H \leq N$ become boundaries of the feasible region? This is a simplified example of what operations researchers call "sensitivity analysis."

Extensions

The four exercises on the following pages extend the base problem. These might be assigned as homework examples at the teacher's discretion. In order to develop a deeper understanding of real-life applications of mathematics, students should be allowed to discuss these extensions, as well as other sorts of situations and the differing constraints they would entail.

The first three extensions are relatively straightforward. The fourth extension discusses the nonlinearity of octane. A graphing calculator is necessary to complete the fourth extension and a bit more structure has been provided to assist students.

Vapor Pressure

Extension 1: Leave all the conditions the same as in the sample problem except for the vapor pressure. Rewrite that problem as "The first grade has an octane number of 92, a vapor pressure of 5.5 psi and contains 0.4% sulfur and the second grade has an octane number of 85, a vapor pressure of 4.5 psi and contains 0.25% sulfur." Leave all of the other characteristics of the problem the same. Solve this new problem.

Reducing the Sulfur Content

Extension 2: Leave all the conditions the same as in the sample problem except the third characteristic of the blend. Rewrite this characteristic as "contains less than or equal 0.34% sulfur." Solve the problem using this new set of constraints.

Increasing the Octane Rating

Extension 3: Leave all the conditions the same as in the sample problem except the first characteristic of the blend. Reword it as "an octane number higher than or equal to 89.1." Solve the problem using these constraints.

An Extension for the Graphing Calculator

Extension 4: Over the years, octane blending studies have shown that the octane number, y_1, of the blend of gasolines in the Jurassic Oil problem is:

$$y_1 = \frac{11{,}040 - 7x}{120} - \frac{2.8x(120 - x)}{120^2},$$

where x = the number (in 1000s) of bbl of Allif gasoline and 120 - x = the number (in 1000s) of bbl of HyOctane gasoline.

1. What should the graph of this function look like? Why?

2. Using a graphing calculator, graph this function with [0,120] x [85,92] as the viewing window.

3. Why does this viewing window make sense for the Jurassic Oil problem?

Did you accurately predict what the graph of this function looks like?

If the behavior of octane numbers were linear, then the function would be $y_2 = \frac{11{,}040 - 7x}{120}$.

4. In the same viewing window, graph y_1 and y_2.

5. Where do these two graphs intersect?

6. Now graph both of these functions using [30,90] x [85,92] for the viewing window.

7. Why does $30 \le x \le 90$ make sense for this problem?

8. For x = 30, 40, 50, 60, 70, 80, and 90, find the difference between y_1 and y_2. Where does it appear that this difference is greatest? Why does that make sense?

9. In the same viewing window, graph y_1 and y_2 along with y_3 = 89, the minimum octane number required of the blend. Then find the point of intersection of the two straight lines.

10. What is the significance of the x-value of this point?

11. Using this x-value, find the value of y_1. What does the value of y_1 represent?

Since x = 51,428.6 bbl of Allif gasoline in the blend actually produces an octane number less than 89, it cannot be the optimal solution to the Jurassic Oil problem.

12. Where in the system of graphs of y_1, y_2 and y_3 is the optimal solution?

13. What is the optimal solution?

14. How much cost was added to the solution found in the base problem, which assumed linearity?

Linear Programming - Real World Blending Examples

CAMP (Computer Assisted Menu Planning)

A study conducted over 50 years ago found that a grown man could subsist on $40 a year for food (August 1939 prices). The resulting meals might not have been very palatable, but they certainly were economical.

Trying to come up with a well balanced meal for a family of four is a difficult task in itself. Trying to come up with well balanced, nutritional, economical meals for patients in a hospital, for prisoners, for school populations, or for nursing homes is very difficult. The menus will also determine equipment and personnel. Hospitals (among other institutions) have constraints on the different limits for fat, cholesterol, sodium, and special diets. Cycles of meals are also a factor in determining the meal to serve. Coming up with a proper menu that keeps cost to a minimum can be costly and time consuming. With the advent of linear programming and the computer, the task has become simplified. CAMP (Computer Assisted Menu Planning) is a program used to help simplify a seemingly overwhelming task. One such program, used in a hospital, had over 30 nutritional constraints, and there were minimum intake levels of 29 nutrients. Also, food preferences needed to be considered. Menu cycles and costs are important. Even color, flavor, and texture can be constraints. An all pale yellow meal (fish, mashed potatoes, squash), while being nutritional and cost effective, may not be appealing. The CAMP program was also able to pick up nutritional requirements for a later meal if a previous meal did not meet the requirements due to other constraints. Though the variables and constraints seemed endless, the program ran successfully and cut food costs by over 10%.

The USDA uses linear programming to set costs for family food plans. This serves as a basis for food stamp allotment.

Lilly Lancaster. The evolution of the Diet Model in Managing Food Systems. *Interfaces 22*:5 (Sept.-Oct. 1992), 59-68.

OMEGA System of Blending Gasoline for Texaco Oil Company

Another use of linear programming is in the OMEGA system of blending gasoline for Texaco Oil Company. Crude oil has different processes it will go through to produce light crude oil such as gasoline, all the way through to heavy crude oil, which would be fuel oil and asphalt. Blending to achieve the different grades of oil is a linear programming problem. Scheduling the blending operations in a plant to produce sufficient amounts of each type of oil is not linear. There are over 15 stocks (such as vapor pressure, octane indices, and lead content) that yield up to eight different blends. There are 40 variables and 71 constraints. By using the OMEGA System in seven of its refineries, Texaco saw a 30% increase in profits, which amounted to over $30 million.

Calvin DeWitt, Leon Lasdon, Allan Waren, Donald Brenner, & Simon Milhem. OMEGA: An Improved Gasoline Blending System for Texaco. *Interfaces, 19*:1 (Jan.-Feb. 1989), 85-101.

Mathematical Models in Farm Planning

The uses for linear programming in solving problems in farming are numerous. It is used in cropping policy, fertilizer blending for maximum efficiency and minimum cost, farm expansion by land purchase and use, and daily harvest operations. Dairy cattle and pig feed formula for optimum weight gain and minimum cost, the types of cattle to be raised, and integrating beef and crop production are just a few of the areas where computer programs have assisted farmers. The National Research Council (US) and the Agricultural Research Council (United Kingdom) have used linear programming to aid the farmers and saved them time and expense.

John J. Glen. Mathematical Models in Farm Planning: A Survey. *Operations Research, 35*:5 (Sept.-Oct. 1987), 641-665.

HOMEWORK PROBLEMS

1. A fruit grower can use two types of fertilizer in his orange grove, Brand A and Brand B. The amounts (in pounds) of nitrogen, phosphoric acid, and chlorine in a bag of each brand are given in the table. Tests indicate that the grove needs at least 1,000 pounds of phosphoric acid and at most 400 pounds of chlorine.

Pounds per Bag	**Brand A**	**Brand B**
Nitrogen	8	3
Phosphoric Acid	4	4
Chlorine	2	1

   If the grower wants to maximize the amount of nitrogen added to the grove, how many bags of each mix should be used? How much nitrogen will be added?

2. A dietitian in a hospital is to arrange a special diet composed of two foods, M and N. Each ounce of food M contains 30 units of calcium, 10 units of iron, 10 units of vitamin A, and 8 units of cholesterol. Each ounce of food N contains 10 units of calcium, 10 units of iron, 30 units of vitamin A, and 4 units of cholesterol. If the minimum daily requirements are 360 units of calcium, 160 units of iron, and 240 units of vitamin A, how many ounces of each food should be used to meet the minimum requirements, and, at the same time, minimize the cholesterol intake? What is the minimum cholesterol intake?

3. Each week, the DeeLite Milk Company gets milk from two dairies and then blends the milk to get the desired amount of butterfat for the company's premier product. Dairy A can supply at most 700 gallons of milk averaging 3.7% butterfat and costing $0.90 per gal. Dairy B can supply milk averaging 3.2% butterfat costing $0.75 per gal. How much milk from each supplier should DeeLite use to get 1000 gallons of milk with at least 3.5% butterfat?

Project Ideas

1. Interview the food service manager at your school or a dietician at a local hospital. Ask about daily nutritional requirements. Find out how menus are planned. Determine if the meals meet the minimum daily requirements. Write a report describing what you learned.

2. Research the recommended daily nutritional allowances for a teenager. Design healthy lunch menus for one week for your school cafeteria.

3. Choose at least five different types of breakfast cereals. Copy the nutritional information given on the side of each box. For each cereal, find the total number of vitamins and minerals that provide at least 25% of the recommended daily allowance. Compare the cereals' sugar and vitamin and mineral content in the form of a poster, article, commercial, etc.

4. Visit a greenhouse and learn how they determine the appropriate fertilizer mixes for all of the different types of vegetables, plants, shrubs, trees, etc. Write a report describing what you learned.

5. Surf the World Wide Web to learn the locations of gasoline blending facilities. List at least six such sites, and indicate the location which is closest to your school. If there is one located relatively close to your school, ask your teacher to help you plan a field trip for your class.

Solutions to Jurassic Oil - Base Problem:

1 How much of each type of gasoline to purchase from each company.

2 \$1,350,000 = (\$15) (90,000)

3 \$600,000 = (\$20) (30,000)

4 \$1,950,000

5 Total Cost = 15A + 20H

6 $A \leq 90{,}000$; $H \leq 90{,}000$

7 A + H = 120,000

8 It is an equation not an inequality.

9 \$16.25 per barrel = \$1,950,000 / 120,000 barrels

10 Answers vary; because this question sets up the concept of a weighted mean, which is used next, you may want to be sure that all of your students understand why the average cost per bbl of the gasoline purchased for the blend is less than the average of the two costs per bbl.

11 $\frac{15A + 20H}{A + H}$

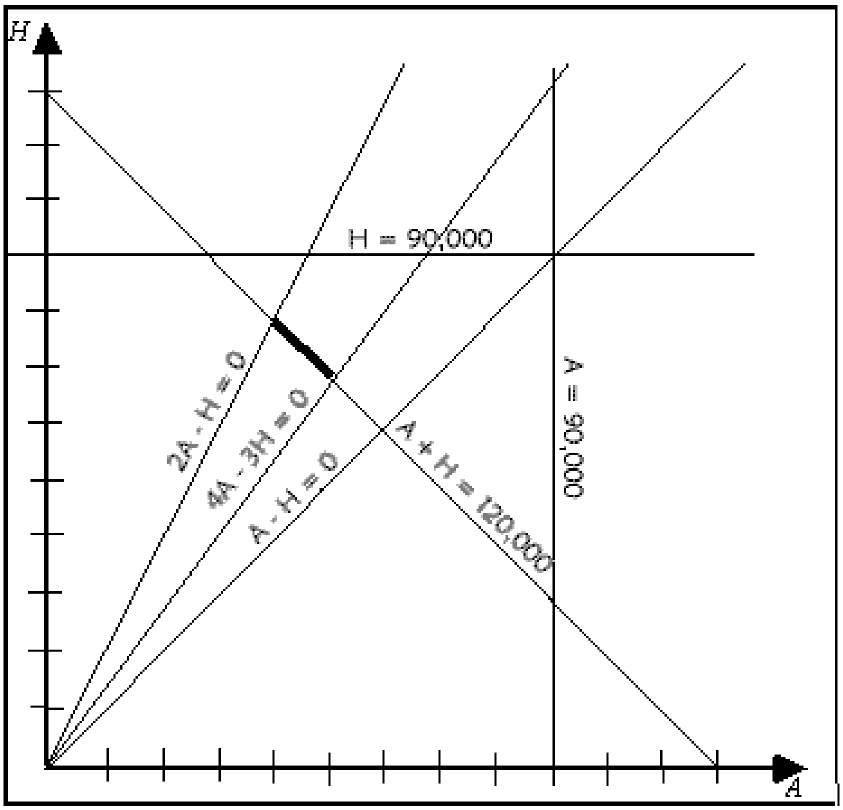
12 5; 5.5; 4.5; $\frac{5.5A + 4.5H}{A + H} \leq 5$

13 sulfur: $\frac{.25A + .4H}{A + H} \leq 0.35$ octane: $\frac{85A + 92H}{A + H} \geq 89$

14 Vapor: $0.5H - 0.5A \geq 0$, or $A - H \leq 0$ Sulfur: $0.05H - 0.1A \leq 0$, or $2A - H \geq 0$ Octane: $3H - 4A \geq 0$, or $4A - 3H \leq 0$

Base Module

Feasible Region and Optimal Solution



15 a line segment

16 2A - H $\geq$ 0 (H $\leq$ 2A); 4A - 3H $\leq$ 0 (H $\leq$ 4/3A); A + H = 120,000

17 A $\leq$ 90,000 ; H $\leq$ 90,000; A - H $\leq$ 0 (H $\geq$ A)

18 Yes, it meets all the constraints.

19 (51,428.6 , 68,571.4) and (40,000 , 80,000)

20 \$2,150,000 = \$15 (50,000) + \$20 (70,000)

21 answers will vary

22 (51,428.6 , 68 571.4) Cost: \$2,142,857 = \$15 (51,428.6) + \$20 (68,571.4)
(40,000 , 80,000) Cost: \$2,200,000 = \$15 (40,000) + \$20 (80,000)

23 (51,428.6 , 68,571.4)

24 The lines are parallel.

25 (51,428.6 , 68,571.4); It is an endpoint of the line segment formed from A + H = 120,000 and two inequalities.

26 51,428.6 barrels from Allif Oil and 68,571.4 barrels from HyOctane will yield the minimum cost.

27 $2,142,857

28 octane number: 88.999998 ≈ 89
vapor pressure: 4.9 psi
sulfur content: 0.34

Solutions to the Extensions

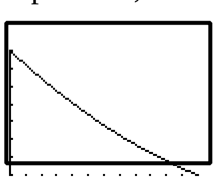
Extension 1: No solution is possible. The "new" vapor pressure constraint: $H - A \leq 0$. The feasible region cannot be drawn to meet **all** of the constraints. When the feasible region is an empty set, there does not exist an optimal solution to the proposed situation. You may wish to have the students sketch the graphs and discuss what could be done in this case.

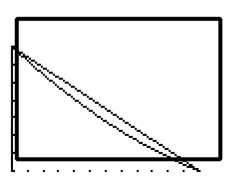
Extension 2: 68,571.4 barrels from HyOctane and 51,428.6 barrels from Allif Oil. This is the same solution as the base problem in the module. The feasible region is a line segment again, but, in this case, the line segment has been shortened. The part of the line that was eliminated did not include the optimal solution of the original problem. Therefore, eliminating that piece from the feasible region does not change the optimal solution.

Extension 3: In this problem, the feasible region is a again a line segment. However, the original optimal solution no longer is part of this line segment. The Octane constraint and the total production constraint now intersect at the point (49,714.3 , 70,285.7). The new total cost would be $2,151,429. This is an increase $8,572 as a result of the 0.1 increase in the Octane requirement.

Extension 4:

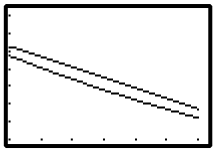
1. A parabola; the function is second degree in x.

2. 

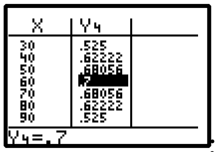
4. 

3. The total number of bbl of gasoline to be blended is 120,000; the octane numbers of the two grades to be blended are 85 and 92.

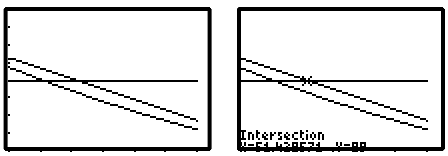
5. (0,92) and (120,85)

6. 

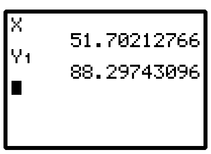
7. For each grade of gasoline, at least 30,000, but not more than 90,000 bbl can be used in the blend.

8. 

; x = 60; if $y_1 = y_2$ at the extremes where none of one of the grades is used in the blend, it makes sense that the greatest difference would occur when the blend uses 50% of each grade.

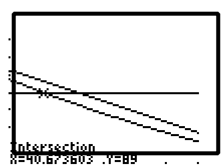
9. 

10. This value of x represents the optimal solution found earlier for the sample problem.

11. 

The true octane number would be about 88.3

12. At the intersection of the horizontal line (y_3) and the parabola (y_1).

13. 

n.b., if x = 40,673.6 bbl are purchased from Allif, then 120 - x = 79,326.4 bbl must be purchased from HyOctane.

14. This non-linear adjustment increases the cost to $2,196,612 as compared to the base problem cost of $2,142,857. This is a net increase of $53,775. This helps explain why the oil industry was concerned about accurately estimating the octane of a blend.

Solutions to the Alternate Scenario:

1 How many barrels to purchase from each supplier.

2 Answers will vary, but the amount purchased from HyOctane should be $\leq$ 90,000 bbl, and the total amount purchased should be $\geq$ 120,000 bbl.

3 Answers will vary with the answer to item #2.

4 \$20(90,000) + \$15(30,000) = \$2,250,000

5 \$15(120,000) = \$1,800,000

6 C = 15 A + 20 H

7 H $\leq$ 90,000

8 A + H $\geq$ 120,000

9 \$2,250,000/120,000 = \$18.75 per bbl

10 \$17.50 is the average of \$15 and \$20. However, that cannot apply to this question because the purchase does not involve an equal number of bbl from each supplier.

11 A + H represents the total number of bbl of gasoline purchased. To find the average cost per bbl, it is necessary to divide by the total number of bbl purchased.

12 5; 5.5; 4.5; $\frac{5.5A + 4.5H}{A + H} \leq 5$

13 $\frac{.25A + .4H}{A + H} \leq 0.35$

14 50,000 + 50,000 < 120,000

15 Answers vary with students' answers to question number 2.

16 (40,000 **,** 80,000); (45,000 **,** 90,000); (60,000 **,** 60,000); (90,000 **,** 90,000)

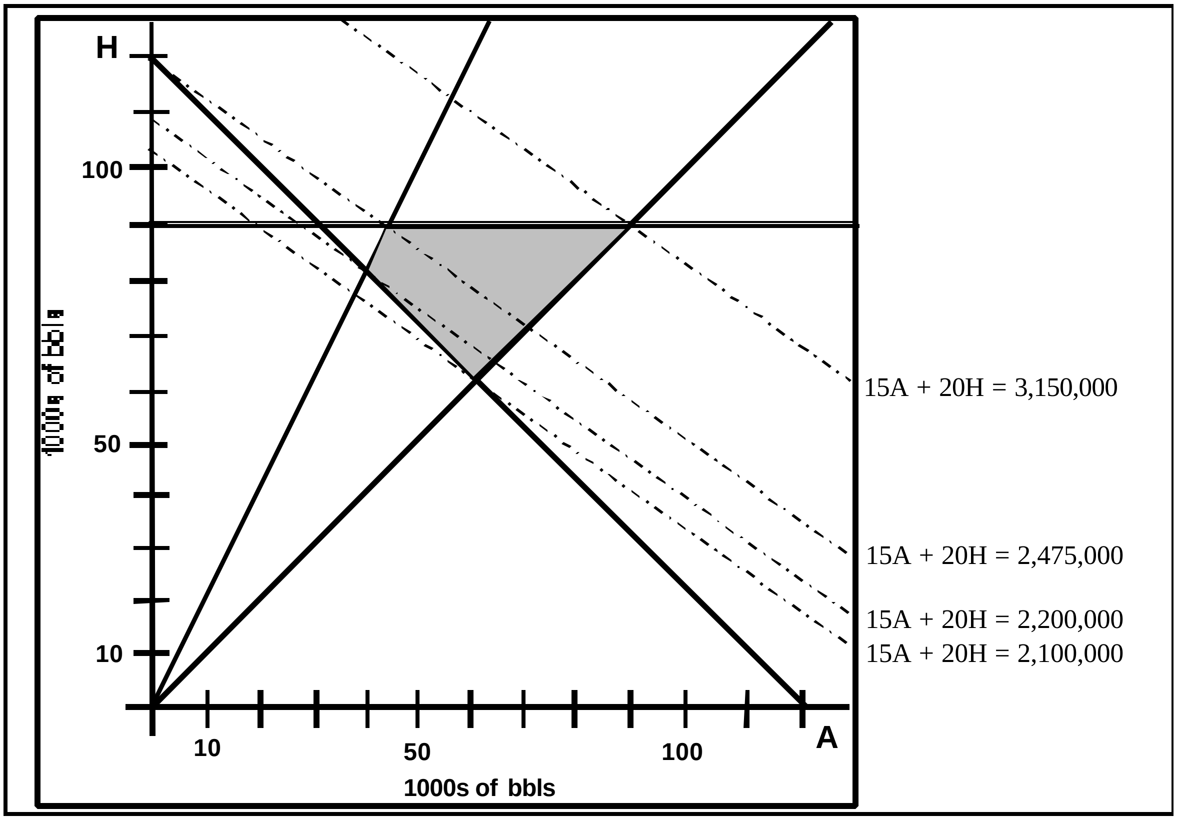
17

A	**H**	**C = 15 A + 20 H**
40,000	80,000	C = \$15(40,000) + \$20(80,000) = \$2,200,000
45,000	90,000	C = \$15(45,000) + \$20(90,000) = \$2,475,000
60,000	60,000	C = \$15(60,000) + \$20(60,000) = \$2,100,000 **
90,000	90,000	C = \$15(90,000) + \$20(90,000) = \$3,150,000

18 (60,000 **,** 60,000)

19 60,000 bbl should be purchased from Allif and 60,000 bbl from HyOctane; the minimum cost is $2,100,000.

20 yes



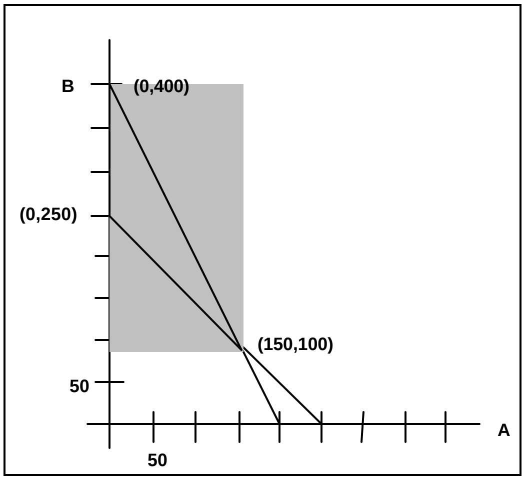
21 The lines are parallel.

22 All of the cost lines form a family of parallel lines. The further left and lower a member of this family is, the lower the cost that line represents. The line with the lowest cost, *while still intersecting the feasible region*, meets all of the constraints. This line intersects the feasible region at (60,000 , 60,000). Therefore, this is the optimal solution.

Solutions to Homework Problems:

1 If N represents the number of lbs. of nitrogen, and A and B represent the number of lbs. of Brand A and Brand B fertilizer, respectively, then the problem can be formulated as:

maximize the objective function, N = 8A + 3B
subject to the constraints 4A + 4B $\geq$ 1000 and 2A + B $\leq$ 400.



Evaluation of Corner Points

N = 8A + 3B
N = 8(0) + 3(400) = 0 + 1200 = 1200

N = 8A + 3B
N = 8(0) + 3(250) = 0 + 750 = 750

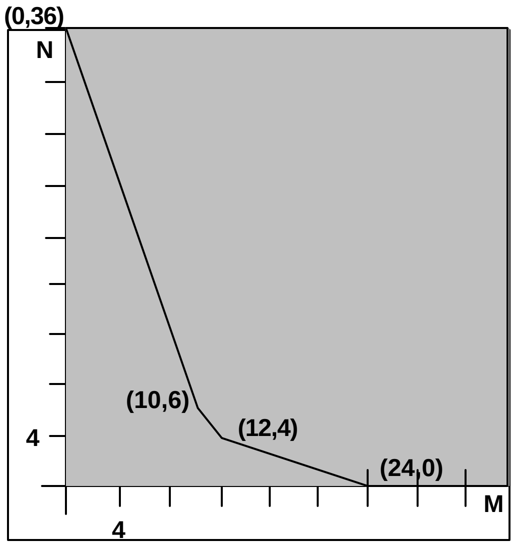
N = 8A + 3B
N = 8(150)+3(100) = 1200 + 300 = 1500

So the optimal solution is (150,100); i.e., 150 lbs. of Brand A and 100 lbs. of Brand B

2 Let C = the number of units of cholesterol and let M and N represent the numbers of oz of food M and food N, respectively. Then the problem can be formulated as:

minimize the objective function, $C = 8M + 4N$, subject to the constraints

$30M + 10N \geq 360$ ($3M + N \geq 36$),
$10M + 10N \geq 160$ ($M + N \geq 16$), and
$10M + 30N \geq 240$ ($M + 3N \geq 24$).



Evaluation of Corner Points

$C = 8M + 4N$
$C = 8(0) + 4(36) = 0 + 144 = 144$

$C = 8M + 4N$
$C = 8(10) + 4(6) = 80 + 24 = 104$

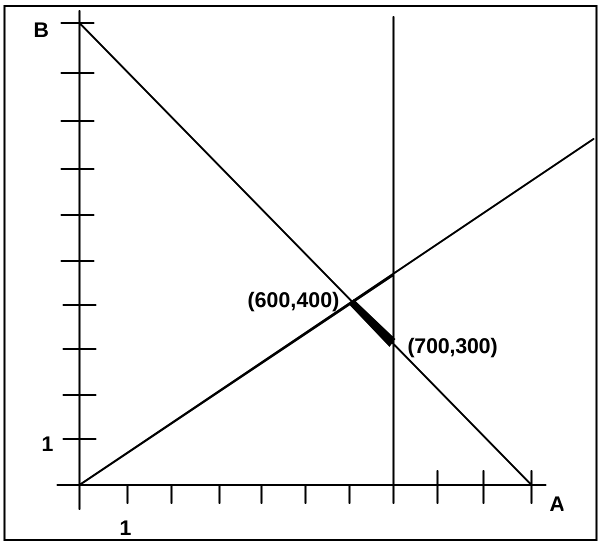
$C = 8M + 4N$
$C = 8(12) + 4(4) = 96 + 16 = 112$

$C = 8M + 4N$
$C = 8(24) + 4(0) = 192 + 0 = 192$

So the optimal solution is (10,6); i.e., 10 oz of food M and 6 oz of food N.

3 Let C = the cost of the milk blend and let A and B represent the number of gal of milk purchased from Dairy A and Dairy B, respectively. Then:

minimize C = 1.5 A + 1.25 B subject to the constraints:
$A \leq 700$
$A + B = 1000$
$\frac{3.7A + 3.2B}{A + B} \geq 3.5 \; (B \leq \frac{2}{3} A)$



Evaluation of Corner Points

Notice that the feasible "region" is the line segment whose endpoints are (600,400) and (700,300).

C = 0.9 A + 0.75 B
C = 0.9 (700) + 0.75 (300)
C = 630 + 225 = 855

C = 0.9 A + 0.75 B
C = 0.9 (600) + 0.75 (400)
C = 540 + 300 = 840

So, the optimal solution is (600,400); i.e., purchase 600 gal from Dairy A and 400 gal from Dairy B. The minimum cost is then $840.