Teacher Resources

Background: Decision Analysis – Decision Trees

We all make decisions in our jobs, our communities and in our personal lives that involve significant uncertainty.

- How much should a company bid on an oil lease and how much should it invest in developing an oil field? On a personal level, how much should I invest in a particular stock or mutual fund?
- How much capacity should be added to the manufacturing plant?
- Should millions be invested in new drug that has proven effective in animal tests?
- Which type of eye surgery should I have?
- Should the next baseball player at bat bunt?

There is uncertainty as to the oil reserves of the oil field and the market demand for a new product. The link between animal drug trials and human effectiveness is far from perfect. What are the risks with the various types of surgery? *Decision analysis* is an operations research modeling tool used to select the best decision in the presence of uncertainty.

The oil industry was one of the earliest users of the tool and continues to lead in its application. Pharmaceutical companies apply decision trees, which are an application of probability trees to make R&D decisions. Industrial giants such as DuPont, Kodak and GM use it to plan new products and capacity. The Decision Analysis Affinity Group (www.daag.org) is an organization that runs conferences at which corporate practitioners of decision analysis share experiences. There is also the Decision Analysis Society which is an organization affiliated with INFORMS. They maintain a homepage at http://faculty.fuqua.duke.edu/daweb/.

The methodology involves creating a probabilistic tree for every alternative. The nodes of a tree represent either decisions or random events. The branches emanating from a node correspond to alternative decisions or alternative outcomes. The best alternative either maximizes the expected profit or minimizes the expected cost. Modern software such as Precision Tree, an Excel add-on, facilitates the analysis and offers graphical representations of the results. These enable a decision maker to explore the strengths and weaknesses of the alternatives.

As decision analysis developed, the leaders in the field recognized two critical psychological and practical issues that needed to be addressed in order to make the tool of greater practical value. The models required estimates of probabilities that were often not readily obtainable through detailed analysis of data. Subject matter experts, therefore, were interviewed in order to estimate the probabilities. Decision analysts, along with mathematical psychologists, became leaders in the effort to understand biases and misconceptions that individuals display when asked to make a forecast. They developed interview protocols to elicit expert opinion in a manner that reduced the likely bias. For example, project managers are often overly optimistic when they forecast how quickly a project will be completed. The interviewer encourages the manager to recall his experiences when projects did not go as planned and to use that relevant experience to make a more realistic forecast for the current project.

The expected value, however, does not capture the fact that people are often fearful of taking risks, especially large ones. This risk aversion is the foundation for all of the insurance industry and the huge market in extended warranties. Decision analysts became leaders in researching attitudes towards risk and designing a methodology called utility theory, that captures this behavior. Utility theory is used to capture the decision maker's risk attitude and incorporate his value system into the decision tree structure.

Case Studies

Drug Tests for Student Athletes

In the spring of 1987, the Athletic Director at Santa Clara University presented a proposal to the university's Athletic Board of Governance to test all student athletes for drug use. Some straightforward techniques of operations research, including a decision tree, were applied to the question of whether to test any single individual for the presence of drugs.

The heart of this analysis was a decision tree which began with the simple decision node having branches "test" and "do not test" and progressed to three outcomes: Drug user identified, false accusation, and unidentified drug user. Tables to determine the probability that a person is a drug user, given that he or she tests positive were developed. Tests having reliabilities between 75% and 99% and possible proportions of drug users in the general population ranging between 5% and 16.6% were included in the tables. The tables of probabilities were then used to determine the reliability that a test would need in order to reduce to an acceptable level the probability of making a false accusation.

Based on this analysis and the ensuing discussion at the Athletic Board of Governance meeting, the Board voted unanimously to recommend to the President of the university not to begin drug testing of student athletes. The Board had simply determined that no available test would reduce the probability of making a false accusation to an acceptable level.

Ultimately, the President of the university adopted the Board's recommendation. The Chairman of the Board later indicated that the analysis of the decision using a decision tree was the prime factor behind the Board's recommendation.

Feinstein, Charles D. "Deciding Whether to Test Student Athletes for Drug Use." *Interfaces* 20:3 (1990) 80 - 87.

Fourth-and-Goal

Toward the end of very close football games, a team is often faced with the decision to kick a field goal to tie the game or go for a touchdown and a win on fourth down. More often than not, the team on offense calls a time-out to discuss the decision.

These discussions usually revolve around whether to go for a touchdown, rather than kick a field goal, and if so, what play to run. Thus, in most cases, the decision to go for a touchdown on fourth down is not made until after deciding what plays to run on first, second, and third down. Hurley (1998) argues that the fourth-and-goal conference should never involve the decision to go for a touchdown, because that decision should be made prior to the selection of the play to run on first down.

The author, who is an assistant football coach at the college level, as well as an operations researcher, came to this conclusion after using a decision tree to analyze such a decision made by his own coaching staff. That decision turned out to be the wrong one, not due to the choice that was made to go for the touchdown, but due to the timing of that decision. The coaching staff all agreed that, had the decision to go for it been made on first down, the sequence of plays selected on first, second, and third down would have been quite different. In other words, if you know that you are playing four-down football on first down, that knowledge affects the particular plays that will be called.

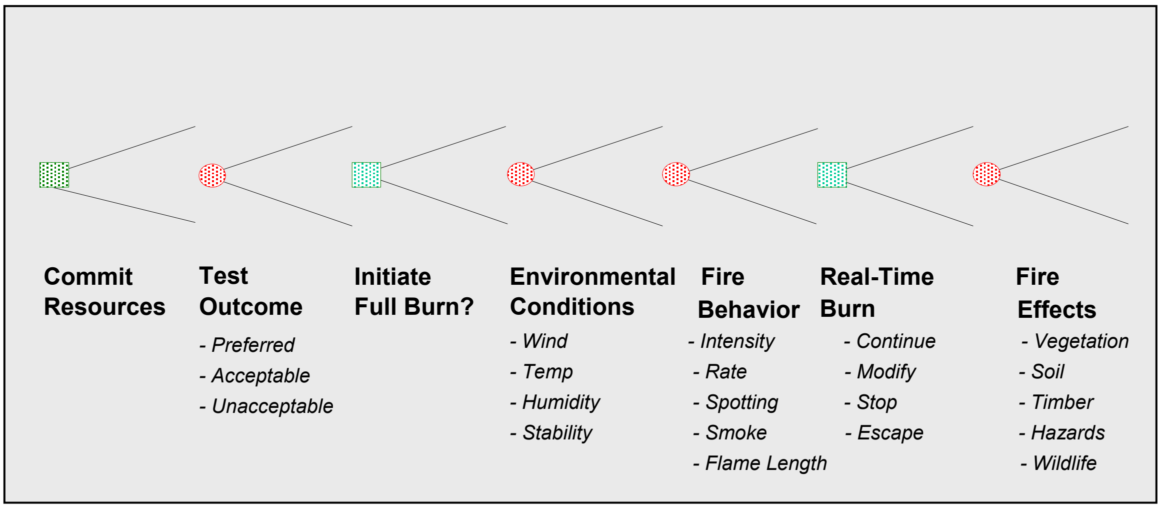
To summarize, in considering a case where a coach decides to go for a touchdown on fourth down, suppose we go back to the decision of what plays to run on the earlier downs and ask the following question: Should the coach choose a sequence of plays that gives him the best chance to score on three downs or on four downs? If he already has made the decision to go for it on fourth down, then the answer to this question is obvious: Choose the sequence that gives you the best chance to score in four downs. This is the primary lesson derived from the analysis of decision trees: The best decision at the present moment depends on best decisions that will be made in the future.

Hurley, William J. "Optimal Sequential Decisions and the Content of the Fourth-and-Goal Conference." Interfaces 28:6 (1998) 19-22.

Using Fire in Forest Management

A fire set under controlled conditions is an important tool in managing the national forests of the United States. These fires are used to clear away forest residue that might otherwise turn a minor fire into a major conflagration. A prescribed burn might be used to clear an area as small as 15 acres in the Tahoe National Forest in Nevada or as large 2000 acres in the Prescott National Forest in Arizona. They are also utilized to enhance wildlife habitat and prepare a site for seedlings. However, planning and executing a controlled fire is a complex and risky process. The spread of a fire is affected by uncertainty surrounding the environmental conditions and the fire's behavior once it is started. Decision makers must decide under what conditions to start a fire and the level of resources to made available on-site as the controlled fire is initiated. The final outcome is also uncertain as to its effects on vegetation, soil, timber, hazards, and wildlife.

Once a fire plan has been established, decision makers still must make a careful assessment of current and forecasted weather conditions before going ahead. The decision elements (rectangles) and the uncertain variables (circles) are represented in a schematic decision tree below. The article describes three case studies in the early 1980s. The primary goals with regard to Tahoe National Forest were to prepare the land for planting new trees and reduce the hazard of wildfire. In the Prescott National Forest the main goal was to improve the wildlife habitat. Another decision tree study was prepared for planting and wildfire reduction in the Gifford Pinchot National Forest that straddles the border between Washington and Oregon.



Cohan, David, Haas, Stephen M., Radloff, David L. and Yancik, Richard F. "Using Fire in Forest Management: Decision Making Under Uncertainty," *Interfaces* 14:5 (1984) 8-19.

Zip + 4

During the early and mid 1980s, the United States Postal Service (USPS) was faced with the decision whether to extend postal automation, and if so, by what means. The Office of Technology Assessment (OTA) was asked to investigate the advisability of proceeding with Phase 2 of the postal automation strategy on both technical and economic grounds. The USPS's strategy was judged to be technically feasible, although a technology other than the USPS's choice was deemed a worthy alternative.

The decision ultimately centered on which technology would perform the best in terms of economic savings. The USPS's choice was for single-line optical character readers, while the proposed alternative was for a multi-line reader which could convert a full address that used only the standard five-digit Zip code to Zip + 4. Both types of reader then place a bar code on the letter, and the bar code is read by an automatic sorter. The advantage of Zip + 4 is that the automatic sorter can then sort the mail to the level of a carrier route, rather than a post office or postal zone. This question was further complicated by uncertainty about the use of ZIP + 4 by consumers. Historically, the USPS had overestimated consumer use of its innovations.

To address this complex and uncertain situation, Decision Science Consortium was contracted to perform a decision analysis of postal automation alternatives. A complex decision tree with six decision branches was developed. Each decision branch except the one for canceling the automation altogether was subjected to a probabilistic analysis of three factors: rate of Zip + 4 usage, savings percentage factor, and usage savings factor. Next, for each path in the decision tree, a detailed cash-flow analysis was developed in order to compare the outcomes of the various alternatives.

The results of this analysis indicated that the NPVs of the five alternatives that would continue the postal automation ranged between $900 million and $1.5 billion. On an expected value basis, all of these options were preferable to canceling the automation, and the option to convert from single-line to multi-line optical character readers was the optimal decision.

Finally, sensitivity analyses were performed to consider the uncertainty in the evaluations. Based on these analyses, the following conclusions were reached:

1. Any continuation of postal automation was better than canceling.
2. Converting to multi-line optical character readers was preferred.
3. Uncertainty about the cost of the multi-line readers contributed very little to the uncertainty of its NPV.
4. Uncertainty about Zip + 4 usage contributed the most to variations in NPV.

Thus, the USPS's main arguments against the use of multi-line readers, that their price and performance were uncertain, were found to be insignificant when compared with other factors, particularly the uncertainty of rate of Zip + 4 usage. This analysis formed the basis of the OTA's report and recommendations to Congress, and the decision was made to convert to multi-line readers. The savings to the USPS (and US taxpayers) was estimated to be $1.5 billion, some $200 million more than had the USPS's first choice been employed.

Ulvila, Jacob W. "Postal Automation (Zip + 4) Technology: A Decision Analysis." *Interfaces* 17:2 (1987) 1-12.

Ulvila, Jacob W. "20/30 Hindsight: The Automatic Zipper." *Interfaces* 18:1 (1988) 74-77.

Objectives of the Module

In this module, students will learn how to structure a decision in a probabilistic environment. The role and limitations of expected value will be explored. Students will use a tree diagram called a decision tree to explore the decision to carry collision insurance on an automobile or not, and if so, the size of the deductible amount to carry. As prerequisite knowledge, students should be familiar with the use of tree diagrams to find the probability of two or more independent events. Alternatively, you could use the two student activities to develop tree diagrams, but doing so will require a somewhat more structured approach to the problem than the one we have provided. Specific student objectives include:

1. Students will compute compound probabilities and calculate expected values of a random variable.
2. Students will construct a decision tree to aid in this analysis.
3. Students will use the expected value of a random variable to make a real-world decision.

Initiating Activity

Since many students may be unfamiliar with the aspects of automobile insurance that are used in the student activity, we suggest opening a pre-activity dialogue with students to explore the following questions. Organizing the students into cooperative discussion groups of three or four at the start of class may facilitate a later whole class discussion.

1. Why do people purchase automobile insurance?
2. What is the difference between liability and collision insurance on an automobile?
3. If a collision insurance policy on an automobile has a $500 deductible, what does that mean?
4. With regard to insurance, what does the word “premium” mean?
5. Everything else being equal, which collision policy would have the higher premium, one with a $500 deductible or one with a $1,000 deductible? How much difference in the six-month premiums do you think there would be.
6. Would it ever make sense not to carry collision insurance on an automobile?
7. How likely do you think it is that a male teenage driver will have an accident in the next six months?

Teaching Notes

During the initiating activity, you may want to open a discussion regarding the cost of an accident versus the risk, or probability of having an accident, compared with the cost of collision insurance. This may also be a good time to discuss with your students the fact that liability insurance is required, while collision insurance is an option.

We obtained some data on accident rates via the Internet. Based on this data, our best estimate of the likelihood that a teen-aged male driver will have an accident during any six-month period is 30%. This estimate is based on national data. Insurance companies, of course, use local data. You may want to discuss this issue with students.

In question 3 of Part 1: Considering Alternatives, we have assumed that Jee Min is legally responsible for an accident to preclude the possibility that the other driver's liability insurance would cover the event.

In question 4 of Part 1, you may need to remind students that the premium amount is part of Jee Min's total cost.

For students who are unfamiliar with tree diagrams, you may need to provide more detail for them as they complete the decision tree in Part 2: Making a Decision with Impact. You may also wish to revisit the concept of independent events at this point in the Student Activity.

The paragraph following question 6 in Part 2 explains how we will create a discrete approximation to a continuous problem situation. You may want to discuss this fact with students and question whether insurance companies would do the same. You might ask what resources insurance companies could bring to bear on such problems that we do not have available for use in the classroom or at home.

You may also want to discuss Jee Min's decision to use the midpoint of each of the first three damage ranges as the representative of that range for the purpose of computing the expected value. If the data were available, some other measure, such as the mean, median, or modal value, would be more appropriate

When question 12 in Part 2 is discussed, it would be a good time to open a discussion of other factors that might influence the decision. Some possibilities include the individual's ability to tolerate risk, which is explored in the extension, or the individual's need for personal transportation.

Extension

The optimal decision for a car worth $6600 was to purchase no collision insurance. However, you may or may not be averse to taking the risk of having no collision insurance. Perform the analysis again for vehicles with different values and insurance premiums given in the table below. Redo the decision tree analysis and decide what level of deductible you would choose in each case.

Car Value	Premiums	
	500 deductible	1000 deductible
3000	660	543
15000	2627	2301
30000	5086	4499

Homework

1. Major league baseball is concerned about the use of steroids by players. Suppose that a blood test to detect the use of steroids is 97% reliable so that if an athlete has used steroids, the test result will be positive 97% of the time. For athletes who have not used steroids, the test results are slightly less reliable and will indicate a negative result 95% of the time. This means that the rate of false negatives is 3% and the rate of false positives is 5%. In an initial voluntary screening, Major League Baseball found 6% usage amongst their players.

   If a major league baseball player tests positive for the use of steroids, what is the probability that he actually used steroids? Answer the following questions in order to determine this probability.

   a.) What do the terms "false-positive" and "false negative" mean?

   b.) Imagine a large arena in which there are 10,000 people just like the athletes in question. How many of the people in the arena would on average use steroids?

   On average how many of them would test positive?

   c.) In this large arena of 10,000 people, how many of them would on average not use steroids?

   On average how many of them would, nevertheless, test positive?

   d.) In total, on average, how many people in the arena would test positive?

   e.) Of all of those people who would test positive, how many actually use steroids?

   How many are actually non-users?

   f.) Use these numbers to estimate the probability that a person in the arena who tests positive actually uses steroids?

2. Suppose the NY Yankees get a runner on first with one out in the late innings of a game in which the score is tied, with Derrick Jeter and Bernie Williams due to bat next, in that order.

   If Jeter attempts to bunt to move the runner to second base, he will be successful 90% of the time. If he swings away, 25% of the time he will advance the runner to second or third base, 5% of the time he will score the runner, 20% of the time he will hit into an inning-ending double-play, and 50% of the time he will leave a runner at first base.

   If Williams bats with a runner on first, he will score the runner 10% of the time. With a runner on second or third, he will score the runner 25% of the time. Otherwise, the inning will end without scoring a runner.

   Make a decision tree to analyze the question of whether the Yankee manager, Joe Torre, should have Jeter bunt.

   Solve the decision tree in order to compare the probability of scoring if Derek Jeter bunts or if he swings away.

Project Ideas

1. Research the specific premium amounts in your area for various deductibles on collision insurance on an automobile with a $6,600 value. This could be done prior to using the module in class, and average premium amounts could be used in place of the premium amounts in the student activities. Students can also obtain similar data for $15,000 and $30,000 priced vehicles. They can plot the premiums versus the vehicle value and see if they discern a pattern.

2. All home appliances are generally purchased with a one-year warranty. For an additional charge, an extended warranty of two to three years can be purchased.

   A. Visit an appliance store to learn the cost of an appliance of your choice (washer, dryer, refrigerator, TV, computer, etc.), the length of its warranty and what the warranty covers, the length and cost of an extended warranty and what it covers.

   B. Based on your own experience or the experience of members of your family, estimate the likelihood of the appliance needing any repair during the extended warranty period. Then estimate the likelihood that the appliance would need a repair that costs one-half or more of its original cost.

3. Talk to your parents or another member of your family regarding the cost of homeowner's or renter's insurance, what such insurance policies cover, and the amount of any deductibles. Ask why they carry such insurance and whether they have ever been paid for a claim. Write a report on what you learned.

4. Research the rate of automobile accident rates by the age of the driver.

Solutions

Student Activity, Part 1: Considering Alternatives

1. *Answers vary.*
2. *The damage amounts may be anything up to $6,600, the total value of the car.*
3. *$400, $500, $500*
4. *$500 + $1250 = $1750.*
5. *Nothing, if the other driver has insurance, or up to his deductible, if the other driver is not insured.*
6. *$1250, the premium amount.*
7. *Nothing, but if he has an accident, his expenses would be the amount of damage to his car.*
8. *$400, $700, $1200*
9. *30%, 70%*
10. *He must have an accident, and the associated vehicle damage must be less than or equal to $500.*
11. *Multiply the two probabilities.*
12. a. Less than or equal to $500: *0.3x0.45 = 0.135*
    b. Greater than $500, and less than or equal to $1000: *0.045*
    c. Greater than $1000, and less than or equal to half the car value: *0.075*
    d. Greater than half the car value and less than or equal to the total value: *0.045*
    e. *No. Jee Min would come out ahead only if he has an accident in which the total damage was greater than $1750 ($500 deductible + $1250 premium). The probability that he has such an accident is less than 12%.*

Student Activity, Part 2: Making a Decision With Impact

1. *The answers to questions 1, 2, 3, 5, all relate to the tree diagram which follows on the next page.*

2. *See tree diagram*

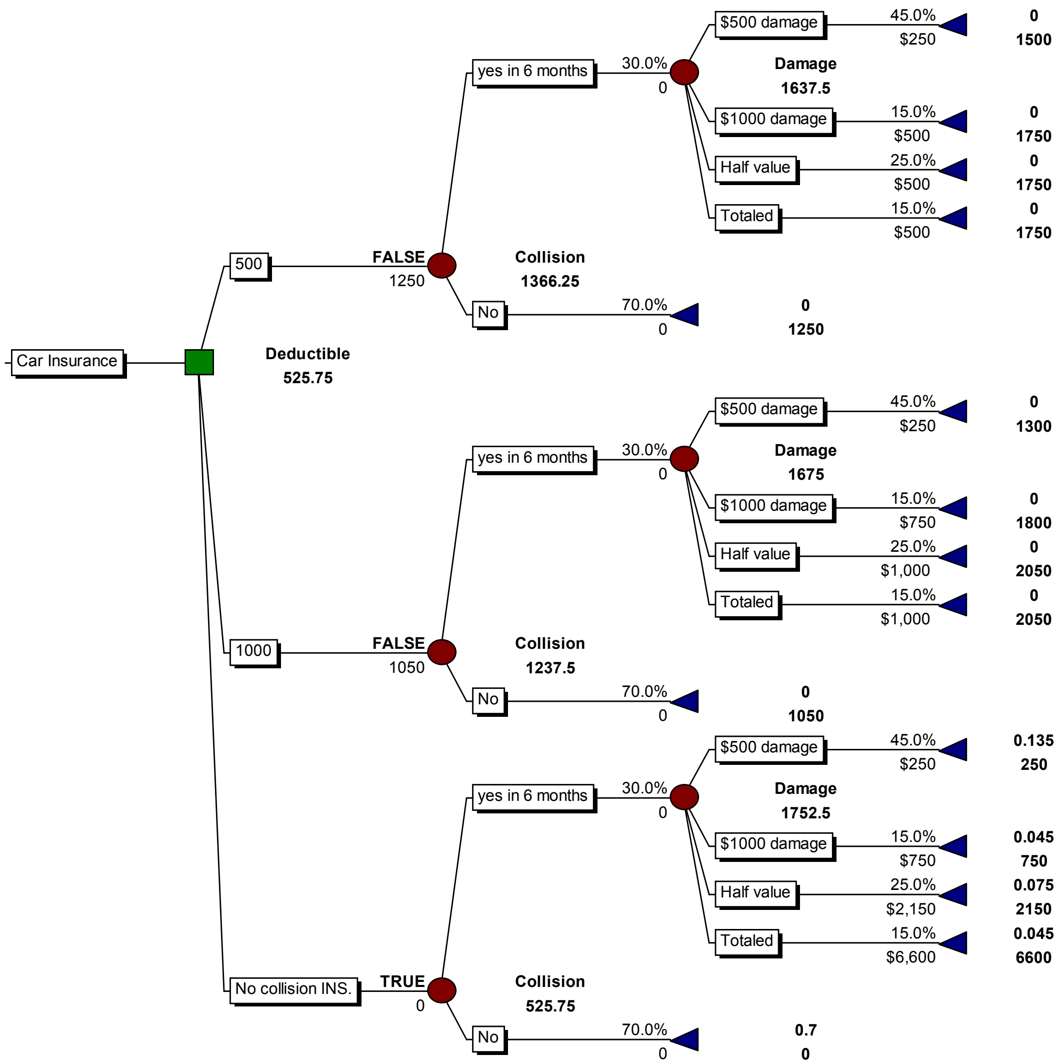
3. *See tree diagram*

4. *15*

5. *See tree diagram.*

6. *0.135+0.045+0.075+0.045=0.3, the probability of having an accident.*

7. *5*



8. *No accident: $1250,*
Damage ≤ $500: $1250 + $250 = $1500,
$500 ≤ Damage ≤ $1000: $1250 + $500 = $1750,
$1000 ≤ Damage ≤ Half value: $1250 + $500 = $1750,
Half value ≤ Damage ≤ Full value: $1250 + $500 = $1750,
and add to tree diagram at the end of each branch sequence.

9. *0.7($1250)+0.135($1500)+0.045($1750)+0.075($1750)+0.045($1750)=$1366.25*

10.

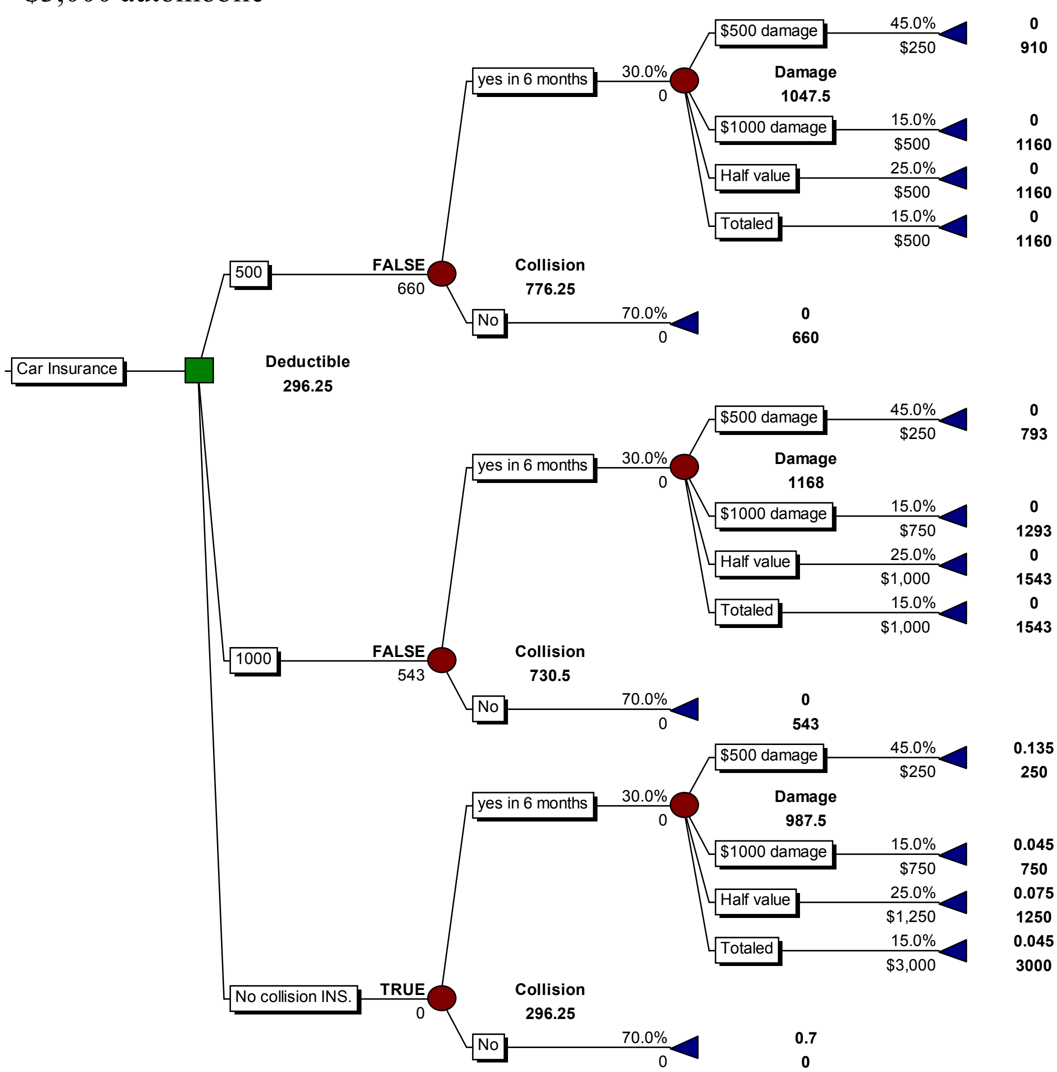
Decision	Expected Cost
No Insurance	*$525.75*
$500 Deductible	*$1366.25*
$1000 Deductible	*$1237.50*

11. *Carrying no collision insurance.*

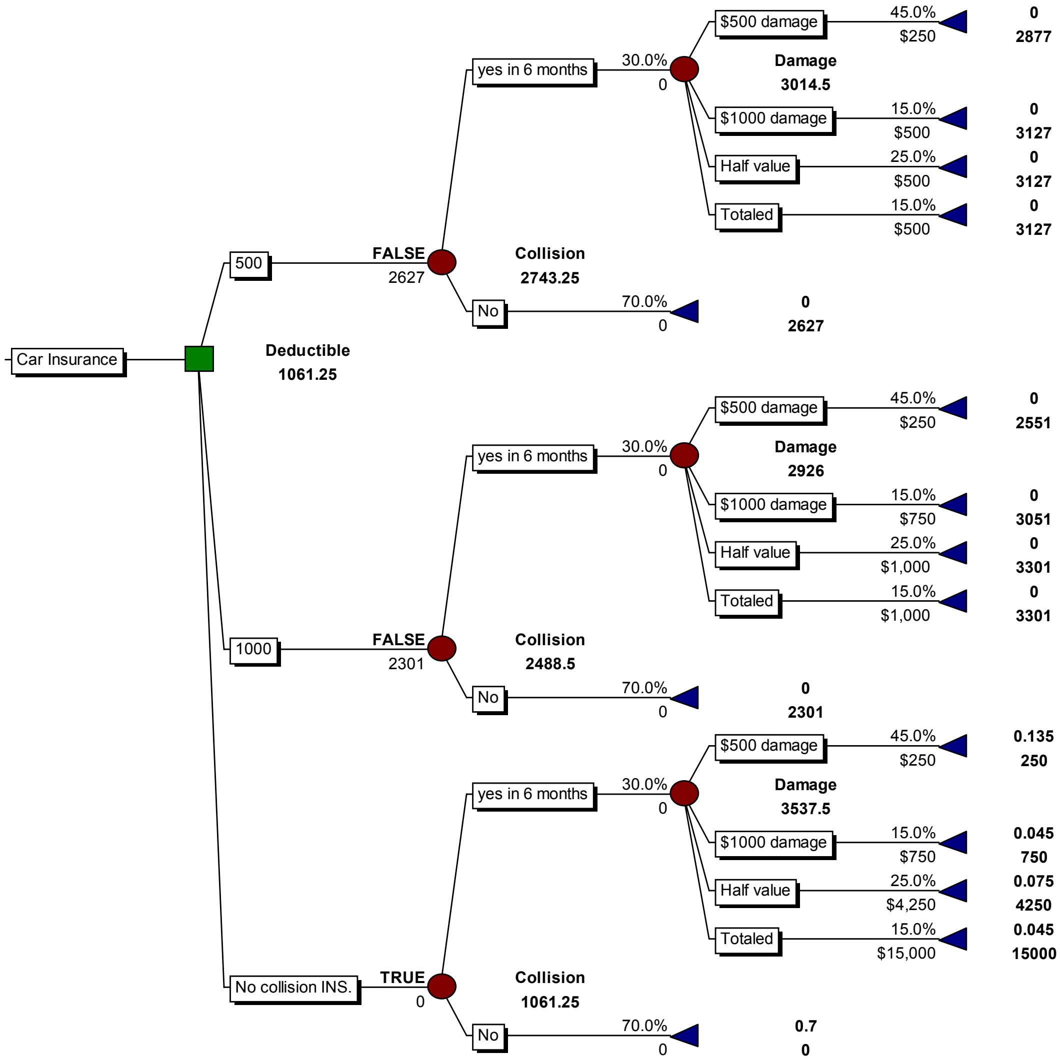
12. *Answers vary, but the decision will depend on many factors such as the ability to tolerate risk or the level of necessity for personal transportation.*

Extension

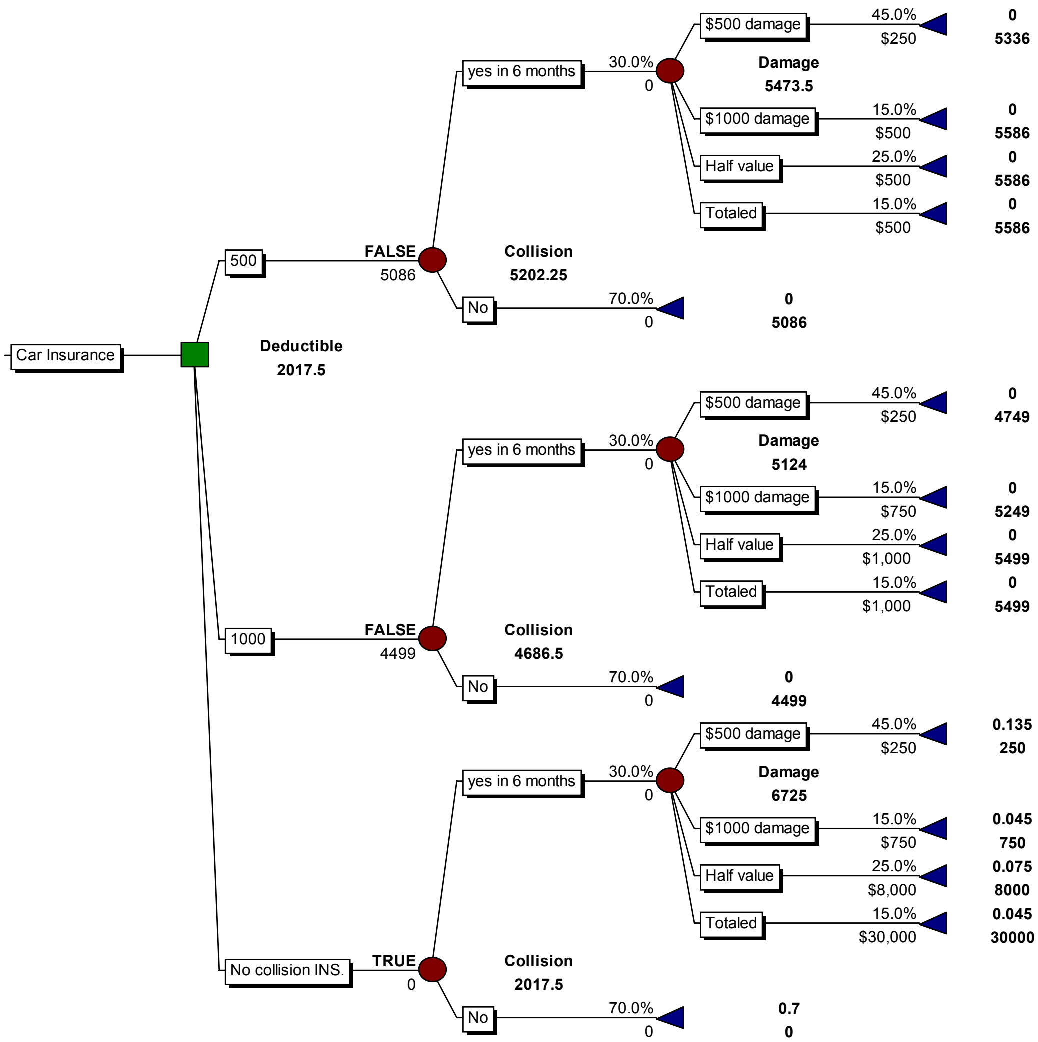
1. $3,000 automobile



2. $15,000 automobile



3. $30,000 automobile



Homework

1. a.) *A false positive occurs when a non-steroid user tests positive. A false negative occurs when a steroid user does not test positive.*

   b.) *6% of 10,000 = 600; 97% of 600 = 582 on average will test positive.*

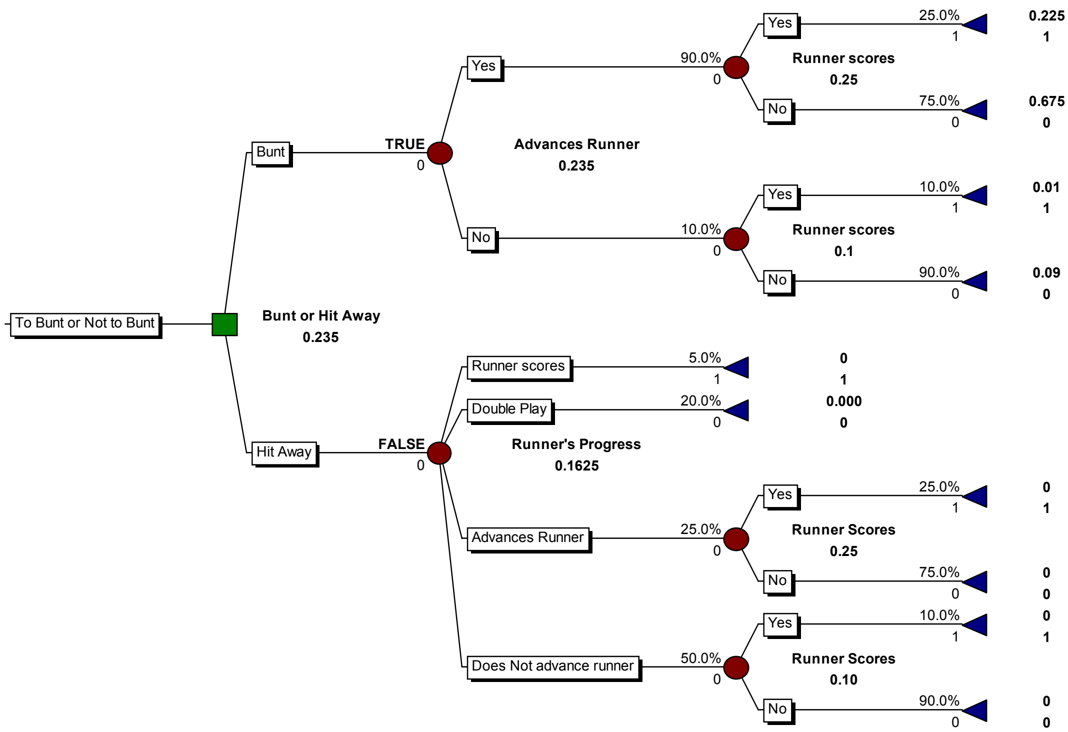
   c.) *94% of 10,000 are on average non-users. Five percent of them, on average will test positive. 5% of 9,400 = 470 on average will (falsely) test positive.*

   d.) *582 + 470 = 1052 people in the arena will, on average, test positive.*

   e.) *On average there are 582 users and 470 non-users who test positive.*

f.) *582/1052 ≈ 55.3%. The dark side of this is that there is likewise approximately a 44.7% chance that someone who tests positive does not use steroids!*

2.



Blackline Masters

Automobile Accident Damage Rates

Amount of Damage	Probability
Less than or equal to $500	45%
Greater than $500, but less than or equal to $1000	15%
Greater than $1000, but less than or equal to one-half the value of the car	25%
Greater than one-half the value of the car, but less than or equal to the total value of the car	15%

Expected Cost of Three Decisions

Decision	Expected Cost
No Insurance	
$500 Deductible	
$1000 Deductible	