Purchasing Collision Insurance – Part 2: Making a Decision with Impact

Jee Min is considering purchasing collision insurance for his 1998 Chevrolet Cavalier, which has a value of $6600. He has learned that the six-month premium amount for collision insurance with a $500 deductible is $1250. He is not sure that he can afford that much, so he decides to investigate the cost of collision insurance with a $1000 deductible. The six-month premium for collision insurance with a $1000 deductible is $1050. He is also considering the possibility of not carrying any collision insurance. In order to make the best decision, Jee Min must consider all of the consequences of each possibility.

1. On a sheet of your own paper, begin to create a decision tree showing the three collision insurance possibilities that Jee Min is considering. Label each branch of your tree.

Recall that according to the National Highway Safety Administration, the probability that a teenage driver in the U.S. will have an accident in any six-month period is 30%.

2. Extend the decision tree you began in #1 by adding a random node at the end of each decision branch. Attach branches to these nodes to account for the possibilities that Jee Min will have an accident or not during a given six-month period.

Label each branch you added in #2 with the probability that this random event occurs.

Recall that if Jee Min does have an accident, the probability of having $500 damage or less to his car is 45%, the probability of the damage amount being greater than $500 and less than or equal to $1000 is 15%, the probability of damage greater than $1000 and less than or equal to one-half the value of the car is 25%, and the probability of the damage amount being greater than one-half the value of the car and less than or equal to the total value of the car is 15%.

3. Extend your decision tree further to show this information. Be sure to label each added branch with the probability of incurring that amount of damage.

4. How many sequences of branches does your tree now have? __________

5. At the end of each sequence of branches, record the probability of that sequence of random events.

6. How can you check your work in #6 to ensure that you have the correct probabilities?

Jee Min realizes that for each complete branch of his tree, he must determine a way to use the probabilities to learn what the cost to him of that branch would be. He would like to use the expected cost associated with each decision. However, he realizes that to do so, he needs
individual damage amounts, not ranges of damage amounts. Jee Min decides to use a single number within each range to represent the damage amount for that sequence of branches. For the three lowest damage ranges, he decides to use the midpoint of the range: $250 for accidents having damage less than or equal to $500, $750 for accidents having damage greater than $500 and less than or equal to $1000, and $2150 for accidents having damage greater than $1000 and less than or equal to one-half the $6600 value of his car. For the last range, damage greater than one-half value and less than or equal to the full value of his car, Jee Min has learned that insurance companies almost always “total” the car when the damage falls within this range. Therefore, Jee Min will use $6600 for accidents in that range.

Consider only the portion of the entire decision tree that represents the decision to purchase $500 deductible collision insurance. Recall that the premium for this deductible amount is $1250.

7. How many sequences of branches fall within this one decision option? _____________

8. For each sequence of branches within the portion of your decision tree that represents the option to purchase $500 deductible collision insurance, calculate the total cost, including the premium amount, of that sequence of branches. Write each total cost at the end of the final branch in each sequence.

9. Use the probabilities and the total costs of the sequences of branches to find the expected cost to Jee Min of the decision to purchase $500 deductible collision insurance.

10. For each of the remaining decision options, in a similar way, calculate the expected cost to Jee Min of making that decision. Enter each of the expected costs in the table below.

<table>
<thead>
<tr>
<th>Decision</th>
<th>Expected Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Insurance</td>
<td></td>
</tr>
<tr>
<td>$500 Deductible</td>
<td></td>
</tr>
<tr>
<td>$1000 Deductible</td>
<td></td>
</tr>
</tbody>
</table>

11. Based on this analysis, which option has the smallest expected cost? __________

12. Should Jee Min base his decision only on this analysis? ___ Explain why or why not.
   ____________________________________________________________________________
### Headlines from the World of Operations Research

<table>
<thead>
<tr>
<th>U.S. Forest Service to Use Fire to Manage Forests</th>
<th>Santa Clara Athletic Board Says No to Drug Testing</th>
<th>U.S. Postal Service Decides Fate of ZIP + 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decision Trees Critical</td>
<td>Santa Clara University’s Athletic Board of Governance voted unanimously to recommend to the President of the University not to begin drug testing of all student athletes. Some straightforward techniques of operations research were applied to the question of whether to test any single individual for the presence of drugs. The heart of this analysis was a decision tree. Tables of probabilities were then developed to determine an acceptable reliability that a test would need in order to reduce to an acceptable level the probability of making a false accusation. Based on this analysis and the ensuing discussion at the Athletic Board of Governance meeting, the Board determined that no available test would reduce the probability to an acceptable level.</td>
<td>Decision Tree Analysis Used</td>
</tr>
</tbody>
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**The Mathematics of Decision-Making in Industry and Government**

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